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Quantised vortex motion through rings in Schrödinger mechanics

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Abstract. Non-relativistic wavefunctions of one-dimensional rings threaded by a magnetic flux are characterised by a quantised vortex structure, which is visualised in the plane into which the ring is embedded. With increasing flux, vortex cores traverse the perimeter of the ring periodically at flux values where anticrossing of the energy levels occurs. The topology of the vortex pattern is related to measurable physical quantities.

1. Introduction

The phase $\varphi(x)$ of complex wavefunctions $\psi(x) = R(x)\exp(i\varphi(x))$, $x \in R^n$, is related through its gradient to wave propagation, particle motion (currents) and, for charged particles, to magnetic properties. In two and higher dimensions the phase gradient may show quantised vortices. This means that its streamlines form closed lines around singularities of the phase function $\varphi(x)$. These singularities occur at zeros of the modulus $R(x)$. In n-dimensional space R^n these zeros are (generically) hypersurfaces of dimension $n-2$. In the plane they are points. On a path enclosing one such phase singularity the phase changes by 2π times an integer.

Quantised vortices have been discussed by Riess (1970) where properties of the phase of stationary Schrödinger functions in the $3N$-dimensional configuration space are investigated from a general point of view and, further, where the special form of the Schrödinger equation is used to relate the nature of the quantised vortex structure to orbital currents and magnetic properties. Hirschfelder and Tang (1976) discuss quantum mechanical streamlines and quantised vortices in atomic and molecular scattering processes, and Hirschfelder (1977) investigates different three-dimensional forms of vortices (see also references quoted in these papers). Kan and Griffin (1977) stress applications to nuclear motion, Nye and Berry (1974) treat the motion of phase gradient vortices associated with solutions of the wave equation (dislocation in wave trains), whereas Berry *et al* (1980) and Berry and Robnik (1986) study two-dimensional systems threaded by a magnetic flux (see § 3).

So far phase gradient vortices have been investigated for wavefunctions which are manifestly spread out in two or higher dimensions. In this paper we consider a one-dimensional multiply connected system: a perturbed loop threaded by a magnetic flux. This represents the simplest case of an arbitrary perturbed network made of one-dimensional branches. On the other hand, such a loop is mathematically equivalent to a rectilinear system with periodic wavefunctions and electromagnetic potentials (see below). We will see that the Schrödinger functions of such one-dimensional periodic

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structures reflect the two dimensionality of the space into which they can be embedded. We will show that for each state of the loop there exists a quantised vortex structure which can be made visible in the two-dimensional neighbourhood of the loop. This two-dimensional vortex structure will be constructed from the calculated values of the phase φ of the Schrödinger function $\psi = R\exp(i\varphi)$ on the one-dimensional branches of the loop. The vortex pattern gives a general flux-dependent characterisation of the state from which it has been constructed. It enables one to understand the properties of the one-dimensional periodic system from a new global point of view.

It is often taken for granted that, for a particle contained in a real multiply connected domain G, the wavefunction must be taken periodic around each loop contained in G (uniqueness of the wavefunction). However, in quantum mechanics wavefunctions must not be single valued *a priori*. Therefore the imposed periodicity of the wavefunction looks like an additional quantum mechanical postulate. But this is not the case in Schrödinger mechanics, since here it can be shown (Riess 1972) that the single-valuedness in G *follows* from the ellipticity of the Schrödinger operator and from the consistency of its definition in the multiply connected domain G with its extension to the two- or three-dimensional (simply connected) space, into which G is embedded and where the Schrödinger operator is generally uniquely defined. Therefore, in Schrödinger mechanics, real one-dimensional loops are mathematically equivalent to rectilinear periodic one-dimensional systems with periodic boundary conditions.

Vortex structures as we will discuss here for a perturbed loop are relevant to most of the multiply connected systems enclosing magnetic flux. Such systems are important in various areas of quantum mechanics either as real systems (e.g. Aharonov–Bohm effects, normal and superconducting networks, cylinders, etc) or rather as mathematical idealisations of physical situations (e.g. periodic boundary conditions in solid state physics). Ring, cylinder or torus geometries are also useful in the theory of electric conductivity, where an electric field is described by a time-dependent magnetic flux (for small electric field the adiabatic approximation of the Schrödinger equation is important, where the flux is a parameter as in the present paper).

2. The energy levels of the perturbed ring

To illustrate the essence of the general behaviour we consider an electron on a one-dimensional ring with circumference $L = L_A + L_B$ (see the first ring shown in figure 1). On the interval L_B a constant potential V is present (representing the simplest form of a perturbation), whereas V is zero elsewhere. The ring is threaded by a magnetic flux ϕ.

Mathematically a network composed of one-dimensional branches is treated branch by branch taking account of the correct branching conditions on the wavefunction at the junctions, which then uniquely determine the Hamiltonian of the network. The Schrödinger equation of an arbitrary network, composed of branches with different constant electric potentials, is formally equivalent to the linear Ginzburg–Landau equation of a superconducting network composed of branches of different materials. The network equation for such an arbitrary superconducting proximity network has been derived by Riess (1983a, equation (15)). This equation can be immediately applied to the special case of our ring if one makes the identifications

$$\theta_A = L_A(2m|E|)^{1/2}/\hbar \tag{1}$$

$$\theta_B = L_B[2m(E-V)]^{1/2}/\hbar \tag{2}$$

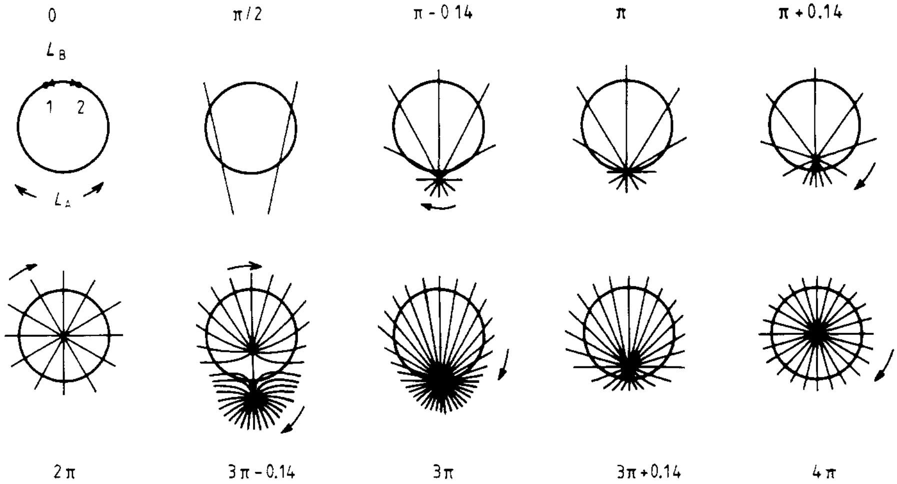

Figure 1. Vortex pattern associated with the lowest energy level as a function of the flux ϕ ($V = -6\times10^{-14}$ erg). The difference between neighbouring lines of constant phase φ is $\pi/6$ and φ increases in the direction of the arrows.

where E is the energy and m is the mass of the electron. The Schrödinger equation of an electron on the ring, expressed in terms of the unknown values ψ_1 and ψ_2 of its solution at point 1 and 2 (figure 1), then takes the form

$$\begin{aligned} -\alpha\psi_1 + \beta\psi_2 &= 0 \\ \beta^*\psi_1 - \alpha\psi_2 &= 0. \end{aligned} \tag{3}$$

Here

$$\alpha = (\theta_A/L_A)\,\widehat{\cot}\,\theta_A + (\theta_B/L_B)\cot\theta_B \tag{4}$$

$$\beta = (\theta_A/L_A)\exp(i\gamma_A)/\widehat{\sin}\,\theta_A + (\theta_B/L_B)\exp(i\gamma_B)/\sin\theta_B \tag{5}$$

$$\widehat{\sin}\,\theta_A, \widehat{\cot}\,\theta_A = \begin{cases} \sin\theta_A, \cot\theta_A & \text{if } E>0 \\ \sinh\theta_A, \coth\theta_A & \text{if } E<0 \end{cases} \tag{6}$$

and $\gamma_{A,B}$ are the line integrals of the vector potential

$$\boldsymbol{A} = (0, 0, B)\times(x, y, 0)/2$$

from point 1 to point 2 on the ring interval L_A, L_B respectively, multiplied by $e/\hbar c$ ($-e$ is the electronic charge).

The compatibility equation for the existence of a solution of the linear system (3) is

$$\alpha^2 - \beta^*\beta = 0. \tag{7}$$

For each flux value ϕ equation (7) has a denumerable set of energy solutions $E_t(\phi)$, $t = 0, 1, 2, \ldots$. $E_t(\phi)$ is periodic with period 2π (ϕ in units of $\hbar c/e$). (In our example one has in addition $E_t(\phi) = E_t(-\phi)$, because the potential $V(s)$ has a reflection symmetry.) These properties can be seen from the explicit form of equation (7). Figure 2 shows the lowest energy curves obtained by solving numerically equation (7) for the parameter values $L_A = 90$ Å, $L_B = 10$ Å, $V = -2.4\times10^{-15}$ erg and $V = -6\times10^{-14}$ erg.

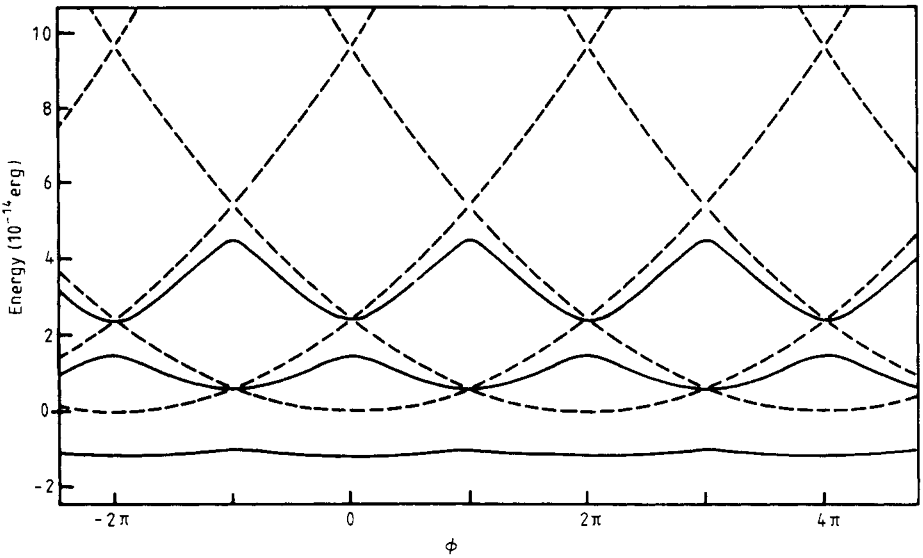

Figure 2. The broken curves show the lowest energy levels of a one-dimensional ring as a function of the magnetic flux ϕ threading the ring (ϕ in units of $\hbar c/e$). The ring is perturbed by a potential well of depth $V = -2.4 \times 10^{-15}$ erg situated on the interval L_B (see figure 1). $L_A = 90$ Å, $L_B = 10$ Å. The parabola-like curves anticross, which is not visible on the scale of the figure. The anticrossing occurs on very short flux intervals around the points $\phi = n\pi$, n integer. Outside these intervals the curves are practically identical with the unperturbed levels ($V = 0$), which are true parabolas intersecting at all the values $\phi = n\pi$. The full curves show the three lowest energy levels for $V = -6 \times 10^{-14}$ erg.

If V is small the form of the curves $E_l(\phi)$ are almost identical with those of an unperturbed ring ($V = 0$), for which $E_l(\phi)$ are parabolas, which mutually intersect at values $\phi = \pi \times$ integer (ϕ in units of $\hbar c/e$). As soon as the perturbation V is different from zero, the curves $E_l(\phi)$ no longer intersect. Instead there is anticrossing at these points (reflecting the Wigner-von Neumann theorem). The bigger the perturbation V the bigger is the energy splitting at the former crossing points $\phi = n\pi$ and the larger is the 'anticrossing interval' $\Delta\phi$ around these points, where the $E_l(\phi)$ curves deviate appreciably from a parabolic form.

3. Calculation of the vortex pattern

In order to study what happens besides the energy behaviour we have calculated the wavefunctions $\psi_A(s)$ and $\psi_B(s)$ on the intervals L_A and L_B as a function of the curvilinear space parameter s (which in both cases is measured from point 1). These functions are

$$\psi_A(s) = \exp(-ie\phi s/\hbar cL)[\beta \,\widehat{\sin}(\theta_A - s\theta_A/L_A) + \alpha \exp(ie\phi L_A/\hbar cL)\,\widehat{\sin}(s\theta_A/L_A)]/\widehat{\sin}\,\theta_A \tag{8}$$

$$\psi_B(s) = \exp(ie\phi s/\hbar cL)[\beta \sin(\theta_B - s\theta_B/L_B) + \alpha \exp(-ie\phi L_B/\hbar cL)\sin(s\theta_B/L_B)]/\sin\theta_B. \tag{9}$$

Equations (8) and (9) follow from (3) and the analogue to equation (14) of Riess (1983). Here θ_A, θ_B, α, β are functions of $E_l(\phi)$ as determined from (7).

For each energy curve $E_t(\phi)$ we have calculated the phase φ of the wavefunction (8) and (9) as a function of the position on the ring and of the flux ϕ ($\psi(s) = R(s)\exp(\varphi(s))$). We have connected values of equal phase on the perimeter of the ring (where this situation occurs) and, quite generally, constructed lines of constant phase in the surrounding of the ring, which are continuous extensions of the phase on the ring into its two-dimensional neighbourhood of the x, y plane. This leads to a structure which contains quantised vortices of the phase gradient field. Each individual vortex is characterised by a non-zero phase winding number m defined as

$$m = (1/2\pi) \oint_C \operatorname{grad} \varphi(x, y)\, \mathrm{d}r. \tag{10}$$

Here C is a closed loop which encircles the particular vortex core once (i.e. the point where lines of constant phase converge from different directions) but no other vortex core. The numbers m are integers as a consequence of the single-valuedness of the wavefunction (see above). At a vortex core the modulus R of the wavefunction vanishes. (These two properties are true for any function $\psi = R\exp(\mathrm{i}\varphi)$, which is sufficiently regular in two (or more) dimensions. For Schrödinger-type functions the regularity is a consequence of the ellipticity of the Schrödinger equation (see the discussion by Riess (1970, 1972) and references quoted therein).

In figures 1, 3 and 4 some characteristic situations are shown. Figure 1 shows the evolution of the ground state. At zero flux no vortex is present, i.e. the phase φ is

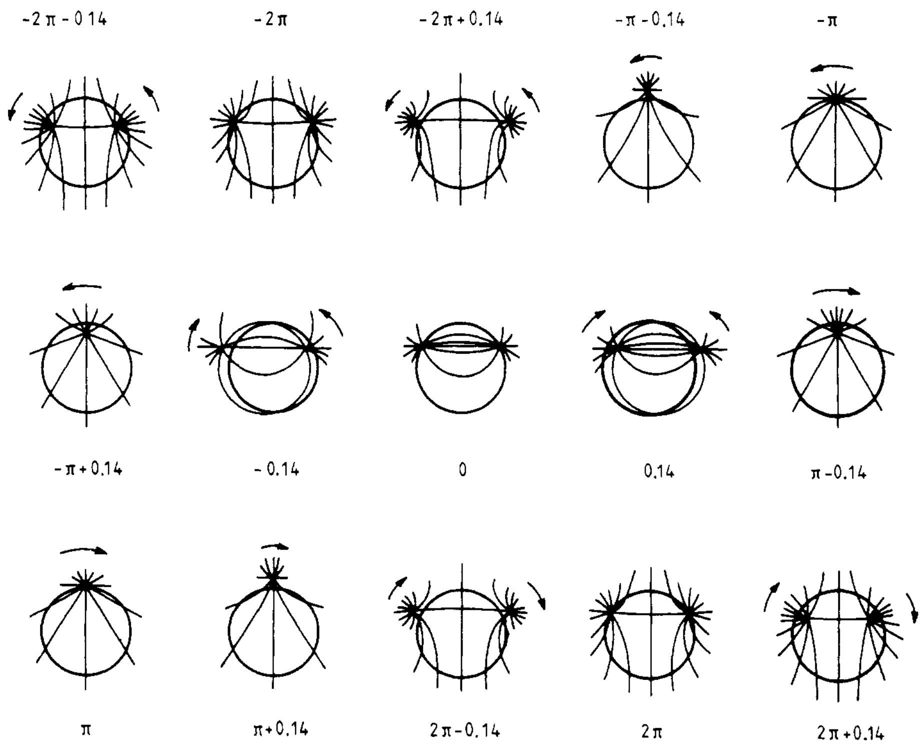

Figure 3. Vortex pattern of the first excited state ($V = -6 \times 10^{-14}$ erg). The difference between neighbouring lines of constant phase φ is $\pi/6$ and φ increases in the direction of the arrows.

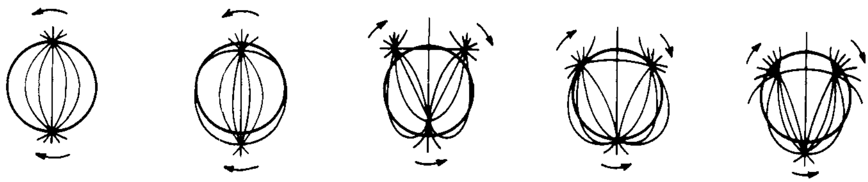

Figure 4. Evolution of the vortex structure of the second excited state ($V = -6 \times 10^{-14}$ erg). The difference between neighbouring lines of constant phase φ is $\pi/6$ and φ increases in the direction of the arrows.

constant everywhere (the wavefunction is real). When ϕ increases a vortex with winding number -1 approaches the ring from the side opposite to the potential well and, for flux equal to π, penetrates into the interior of the ring at the point $s = \frac{1}{2}L_A$. If ϕ is further increased a second vortex of winding number -2 approaches from below, whereas the inner vortex moves 'downwards' towards the perimeter. At $\phi = 3\pi$ both vortices cross the perimeter at $s = \frac{1}{2}L_A$, i.e. here the two vortex cores are united. For $3\pi < \phi < 4\pi$ the $(m = -2)$ vortex is now inside and moves towards the centre of the ring, whereas the $(m = -1)$ vortex is outside and moves away from the ring. The phase winding number along the perimeter is now equal to -2. If ϕ is further increased additional periodic exchanges of vortices occur at $\phi = \pi p$, p odd integer. Each time a vortex with winding number $-\frac{1}{2}(p+1)$ enters and one with winding number $-[\frac{1}{2}(p+1)-1]$ leaves the ring at $s = \frac{1}{2}L_A$. Simultaneously the wavefunction vanishes at this spatial point. (This illustrates the fact that R is periodic with ϕ but φ is not.)

For excited states the vortex pattern is more complex. Figure 3 shows the evolution for the first excited state. Here, at zero flux, one vortex core $(m = -1)$ enters and one $(m = 1)$ leaves the ring at different points of the perimeter. This means that the two nodes of the wavefunction at zero flux (which is real) are phase vortex centres and the lines of constant phase associated with this real function have to be drawn as in figure 3 in order to be consistent with those at ϕ values different from zero into which they continuously develop. Here as well as for all excited states the crossing of vortex cores through the circumference occurs at all flux values equal to π times an integer. These are the flux values where the unperturbed ($V = 0$) energy curves intersect (figure 2).

For the second excited state (figure 4) at $\phi = 0$ two vortex cores with opposite equal winding number $(m = 1$ and $m = -1)$ are present on the circumference. If the flux increases the lower vortex $(m = -1)$ leaves the ring whereas the upper vortex $(m = 1)$ completely enters and gradually moves through the centre towards the point $s = \frac{1}{2}L_A$. Simultaneously two new $(m = -1)$ vortices approach from above. At $\phi = \pi$ all three vortex cores are on the perimeter.

There are two different sets of vortex crossing points on the circumference: those for flux equal to π times an even integer and those for flux equal to π times an odd interger. In the vortex pattern for the first excited state there are always two distinct vortex cores on the circumference for $\phi = n\pi$, n even, and one vortex core 'at the top' of the ring for $\phi = p\pi$, p odd (R is periodic as a function of ϕ with period 2π).

The number of vortex crossing points is related to the orthogonality of the states $\psi_t(s)$. At ϕ equal to π times an even integer the points, where the vortex cores cross the circumference, are equal to the nodes of the functions $\psi_t(s)$ at zero flux (at $\phi = 0$

the $\psi_l(s)$ are real since the Hamiltonian is real in our gauge and since there is no degeneracy). This means: no node for the ground state (and hence no vortex core on the ring perimeter), two nodes for the first excited state (and hence two vortex cores on the ring), two different nodes for the second excited state (and hence two vortex cores at these two different nodal points), two different sets of four nodes for the third and fourth excited state respectively, and so on. Half of these vortex cores enter and half of them leave the ring when ϕ becomes different from zero. Figure 4 illustrates this behaviour for the second excited state.

Further, at ϕ equal to π times an odd integer the sum of the number of nodes of each of the two wavefunctions having developed from a pair of split energies at $\phi = 0$ (or at $n\pi$, n even) is equal to twice the number of nodes of either one of the two functions at $\phi = 0$ (or at $n\pi$, n integer). For instance, the first and second excited state each have two nodes at zero flux. At $\phi = \pi$ the first excited state has one node (i.e. one vortex core on the ring) but the second excited state has three nodes (vortex cores on the ring), i.e. the total number of nodes (vortex cores) of the two states is always four for ϕ equal to π times an integer.

On points off the perimeter line the local form of the lines of constant phase is arbitrary. However, their relevant topological structure, i.e. the number of vortex cores with their individual winding numbers and their positions (inside, outside or on the perimeter) are determined by our extrapolation prescription. Figures 1, 3 and 4 were drawn in such a way that a continuous transition between different vortex patterns at different flux values is obtained, that the pattern is consistent with the calculated phase values on the ring perimeter and that there is spatial continuity in two dimensions (except of course at the vortex centres, where φ jumps). For all flux values ϕ of each of the intervals $n\pi < \phi < (n+\frac{1}{2})\pi$ and $(n-\frac{1}{2})\pi < \phi < n\pi$ we kept the same topological structure, which was obtained for ϕ values close to $\phi = n\pi$, n integer. Here the vortex cores were found to be isolated points with well defined winding numbers. When ϕ tends to $n\pi$, the vortex cores join the perimeter continuously. Therefore for $\phi = n\pi$, all lines of constant phase were drawn converging to these cores on the perimeter (although here the lines outside the perimeter could also be drawn as closing together on a single line (a degenerate vortex core) connecting the perimeter with infinity).

A true gauge transformation (i.e. where f is single valued in the entire plane)

$$\boldsymbol{A}' = \boldsymbol{A} + \operatorname{grad} f(x, y)$$

does not change the topology of the pattern, since the additional phase factor of the transformed wavefunction does not change the phase winding number and the position of the vortex cores.

Quantised vortex structures of truly two-dimensional systems threaded by a magnetic flux have been investigated in two interesting papers: Berry *et al* (1980) studied the Aharonov-Bohm effect from this point of view, whereas Berry and Robnik (1986) considered a charged particle in an asymmetric two-dimensional domain D threaded by a single line of magnetic flux. It was found that the phase winding number W around the area containing the magnetic flux increases by unity each time the flux ϕ increases by 2π. Our calculations confirm and further illustrate this general result. (For our one-dimensional perturbed ring this relation can, for example, be derived from the general ϕ dependence of the vortex patterns.)

In the quoted references the jump of W occurs at ϕ values equal to π times an odd integer and is caused by a single vortex with winding number one, which has migrated from the edge of D into the flux containing domain. This is exactly analogous to what happens in the ground state of our ring system (figure 1). In the examples calculated by Berry and Robnik (1986) the rest of the quantised vortices present in D are not involved in the change of W. This point is different in our one-dimensional loop: here, for excited states, several vortex cores are simultaneously involved in the change of W, leading to jumps of W by more than unity. Further, the jumps of W occur also at ϕ equal to π times an even integer.

4. Discussion

In the flux intervals between adjacent energy anticrossing zones around the flux values $n\pi$, n integer, the wavefunctions $\psi_t(s)$ resemble very much the unperturbed ($V=0$) functions

$$\psi_k(s) \sim \exp(-ik2\pi s/L) \tag{11}$$

which give rise to the (intersecting) parabolic energy curves of figure 2. The smaller the perturbation V the bigger the resemblance in these flux intervals. In terms of vortex structure this means that the flux interval $\Delta\phi$, over which most of the central zone of the vortices crosses the ring (i.e. is situated in the immediate vicinity of the perimeter) is small if V is small (and vice versa). In our examples this crossing zone $\Delta\phi$ is of the order of 0.3 rad for $V=-6\times10^{-14}$ erg and of the order of 0.03 rad for $V=-2.4\times10^{-15}$ erg.

In these anticrossing intervals $\Delta\phi$ around $\phi=n\pi$, n integer, the wavefunctions are very different from the plane waves (11). At $\phi=n\pi$ the phases are discontinuous along the ring across the nodal points, where jumps of multiples of π occur. (However $\psi(s)$ itself remains continuous.)

In the limit, where V tends to zero, the phase functions $\varphi(s)$ change discontinuously with ϕ, i.e. in this limit the k values jump by integers at $\phi=n\pi$, which can be read off figure 2. For the ground state, for example, the k values decrease by one each time ϕ increases by 2π starting at $\phi=\pi$ (see figure 2). This means that in the limit of vanishing perturbation the physically significant wavefunctions are not the ϕ-independent plane waves (11), but are ϕ-dependent functions associated with the anticrossing of the energy levels in the limit $V=0$.

Superconducting micronetworks in the vicinity of the transition temperature can be described by the linear Ginzburg-Landau equation, which is of Schrödinger type. Hence the vortex pattern must show the same characteristics as for electronic loops of similar geometry. However, in superconducting networks only the ground-state solution is of physical significance (corresponding to the highest possible transition temperature, see, e.g., Riess (1983a) and references quoted therein). We have calculated the phase vortex pattern for a loop with an arm (the arm is the equivalent to the perturbation V in the ring discussed above). Here vortices enter and leave the loop periodically at the point opposite to the arm (in the same way as for the ground state of the electron on the ring shown in figure 1). Physically this means that at this spatial point the superconducting network becomes normal periodically with ϕ.

The current around the ring is given by

$$j \sim R^2(s)[(\hbar c/q)(\partial/\partial s)\varphi(s)+A_{\parallel}(s)]$$

($q=e$ for electrons, $q=2e$ for a superconductor, $A_{\parallel}$ is the tangential component of the vector potential). Since $\operatorname{div} j=0$, the current j is the same on each point of the loop. It follows that, where R is small, $\partial\varphi/\partial s$ is large and vice versa. Hence in superconducting loops (or in the ground state of electronic loops) the vortex cores cross the perimeter at the weakest point of the loop (i.e. where $V(s)$ is highest and hence R is small for all ϕ). A high potential barrier ('weak link') represents a large perturbation of the ring. Hence the flux interval $\Delta\phi$ during which a vortex core is pinned to the neighbourhood of its crossing point (which is situated in the barrier) is large, as we have seen. If the height of the barrier tends to infinity the crossing intervals tend to their maximum value 2π (ϕ in units of $\hbar c/q$). This can be shown to lead directly to Josephson effect behaviour.

The systems discussed so far have been aperiodically perturbed rings. They show anticrossing of the energy curves at *all* values $\phi = n\pi$, n integer, where the unperturbed energy parabolas intersect (figure 2). All the possible characteristic scenarios of vortex crossing through a ring can be seen from an aperiodically perturbed ring, because here all the energy levels anticross. If the perturbation is periodic along the ring circumference (i.e. with period L/p, $p>1$, integer) only the levels at the band edges anticross and hence give rise to vortices crossing the perimeter. For these levels the change of the vortex pattern takes place according to the same scenario as for the corresponding levels of the aperiodically perturbed loop.

As an example we have calculated the vortex pattern of a superconducting ring with two equal arms at opposite points of the ring. (This is the analogue to an electron on a ring with two equal potential wells situated at opposite intervals of the ring.) Here the eigenvalues as a function of ϕ cross at $\phi = n\pi$, n odd, and anticross at $\phi = r\pi$, r even. At $\phi = 0$ the first excited state shows two nodes at opposite points symmetrically between the arms. At these nodes one vortex core enters and one leaves the ring similar to the scenario of figure 3. At $\phi = 2\pi$ two ($m = -1$) cores enter simultaneously at the same nodes similar to figure 3. (We remark again that for superconductors only the lowest branch of the eigenvalues has a physical meaning and that ϕ is taken in units of $\hbar c/2e$.) Preliminary results on quantised vortex motion through superconducting rings have been reported earlier (Riess 1983b).

The local value of the phase gradient does not have a direct physical meaning, since it depends on the gauge of the vector potential. On the other hand, the global behaviour of the phase in space, i.e. the topology of the vortex pattern (which is gauge invariant), is directly related to measurable physical quantities. In this context we quote two general theorems (Riess 1970) which are valid for any spin-free system which can be enclosed in a simply connected domain with no current flowing through its boundary: (a) if no vortex core of the phase gradient is present in this domain, the system is diamagnetic and (b) if the system is paramagnetic, there must be at least one vortex core in this domain.

These theorems can be illustrated by the ring systems discussed in the present paper. Consider the ground state for $-\pi < \phi < \pi$. Here there are no vortex cores in the simply connected domain of the system, i.e. inside the perimeter of the ring, as we have seen. Hence it is diamagnetic according to theorem (a). This is in fact the case since $\partial E(\phi)/\partial|\phi| > 0$. Further, when $\pi < \phi < 2\pi$, we have $\partial E/\partial\phi < 0$, i.e. paramagnetism (which can be measured experimentally). Hence, according to theorem (b), there must be a phase gradient vortex core inside the perimeter of the ring. This mathematical property of the wavefunction can thus be predicted from an experimental property of the system. Our explicit calculation (figure 1) confirms this prediction.

Further, quite generally, for any state of the loop system one sees that, whenever there is a change of the phase winding number around the loop (i.e. whenever vortex cores cross the perimeter) there is a change between diamagnetic and paramagnetic behaviour. This again shows that the topology of the vortex structure is directly related to measurable properties of the system.

In this paper the nature of phase vortex patterns associated with one-dimensional periodic structures and their change with magnetic field has been illustrated by considering a perturbed loop. The important feature is the crossing of quantised vortices through the loop at flux values where the energy levels anticross. (This gives a fluid dynamic description of level anticrossing.) The role of phase vorticity of more complex periodic systems will be discussed in further papers.

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